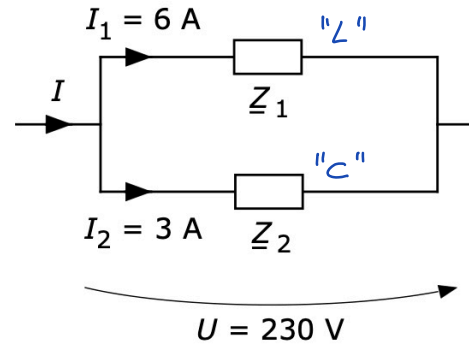


Question 5 (Puissances AC) - Corrigé

$$P_1 = 415 \text{ W} ; X_1 > 0 \quad \underline{Z}_1 = ?$$

$$P_2 = 415 \text{ W} ; X_2 < 0 \quad \underline{Z}_2 = ?$$



1)

$$\text{Pour } R_i : P_i = R_i \cdot I_i^2 \quad (i=1,2)$$

$$R_1 = \frac{P_1}{I_1^2} ; R_1 = \frac{415}{6^2} = 11.528 \, \Omega \quad R_2 = \frac{P_2}{I_2^2} ; R_2 = \frac{415}{3^2} = 46.111 \, \Omega$$

$$\text{Pour } X_i : Q_i = X_i \cdot I_i^2 = U \cdot I_i \sin \varphi_i$$

$$\text{Sachant que } P_i = U \cdot I_i \cdot \cos \varphi_i \rightarrow \varphi_i = \arccos \left(\frac{P_i}{U I_i} \right)$$

$$\text{Il vient : } X_i = \frac{U}{I_i} \sin \left(\arccos \left(\frac{P_i}{U I_i} \right) \right)$$

$$X_1 = \frac{230}{6} \sin \left(\arccos \left(\frac{415}{230 \cdot 6} \right) \right) = +36.56 \, \Omega ; X_2 = -61.25 \, \Omega$$

$$\underline{Z}_1 = 11.528 + j 36.56 \, \Omega = 38.33 \cdot e^{j 72.5^\circ} = 38.33 \cdot e^{j 1.265} \, \Omega$$

$$\underline{Z}_2 = 46.111 - j 61.25 \, \Omega = 76.67 \cdot e^{-j 53.026^\circ} = 76.67 \cdot e^{-j 0.925} \, \Omega$$

2) Calcul de I (méthode puissances) :

$$S = UI \quad \text{avec} \quad S = \sqrt{(P_1 + P_2)^2 + (Q_1 + Q_2)^2}$$

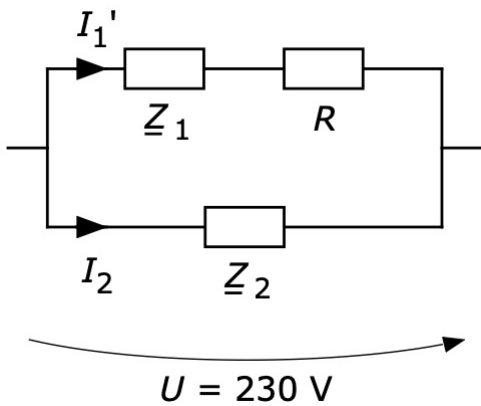
ou: $Q_i = X_i \cdot I_i^2$

$Q_1 = 36.56 \cdot 6^2 = 1316.1 \text{ var}$

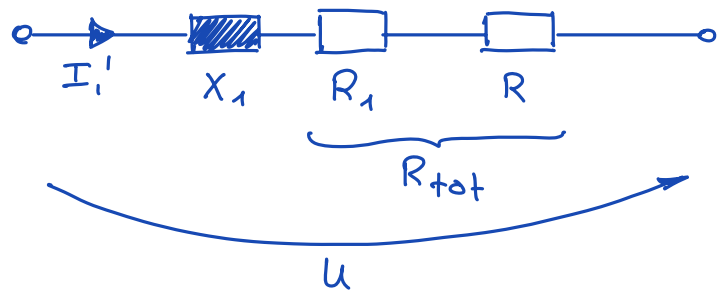
$Q_2 = -61.25 \cdot 3^2 = -551.25 \text{ var}$

$$I = \frac{S}{U} = \frac{1}{230} \sqrt{(415 + 415)^2 + (1316.1 - 551.25)^2} = \frac{1128.69}{230} = 4.907 \text{ A}$$

3)



Branche supérieure: $P' = 295 \text{ W}$



$$I_1' = \frac{U}{Z_1'} \quad \text{ou} \quad I_1'^2 = \frac{U^2}{Z_1'^2} \quad ; \quad P' = R_{\text{tot}} \cdot I_1'^2 \quad (1); (2).$$

4) De (1) et (2) : $R_{\text{tot}} = \frac{P'}{I_1'^2} = \frac{P' Z_1'^2}{U^2} = \frac{P' (R_{\text{tot}}^2 + X_1'^2)}{U^2} \rightarrow$

$$U^2 \cdot R_{\text{tot}} = P' \cdot R_{\text{tot}}^2 + P' \cdot X_1'^2 \quad \text{on} \quad \underline{P' \cdot R_{\text{tot}}^2 - U^2 \cdot R_{\text{tot}} + P' X_1'^2 = 0}$$

A.N.: $295 R_{\text{tot}}^2 - 230^2 R_{\text{tot}} + 295 \cdot 36.56^2 = 0$

Deux solutions, R_{tot}' et R_{tot}'' :

$R_{\text{tot}}' = 7.792 \, \Omega \rightarrow R' = R_{\text{tot}}' - R_1 ; \quad R' = -3.735 \, \Omega \text{ impossible}$

$R_{\text{tot}}'' = 171.53 \, \Omega \rightarrow R'' = R_{\text{tot}}'' - R_1 ; \quad \underline{R'' = 160 \, \Omega}$

2)' Somme des courants (complexe):

$$\underline{I}_1 = \frac{\underline{U}}{\underline{Z}_1} = \frac{230}{38.33 \cdot e^{j 72.5^\circ}} = 6.001 \cdot e^{-j 72.5^\circ}$$

$$\underline{I}_2 = \frac{\underline{U}}{\underline{Z}_2} = \frac{230}{76.67 \cdot e^{-j 53.03^\circ}} = 3.013 \cdot e^{+j 53.03^\circ}$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 6.001 \cdot e^{-j 72.5^\circ} + 3.013 \cdot e^{+j 53.03^\circ}$$

$$= 4.907 \cdot e^{-j 42.52^\circ}$$

$$\Rightarrow \underline{I} = \underline{4.9} \text{ A}$$